

K22P 1602

Reg. No. : .....

Name : .....

I Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022  
(2019 Admission Onwards)

MATHEMATICS

MAT1C02 : Linear Algebra

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Find a basis for a Vector Space  $V = \{(x, y, z) \in \mathbb{R}^3 / y = z + x\}$ .
2. Let  $F$  be a Field and let  $T$  be a operator on  $\mathbb{R}^3$  defined by  $T(x, y, z) = (x + 2y, x + y + z, 2y + 4z)$ . Find the Matrix of  $T$  with respect to standard basis.
3. Show that similar matrices have same characteristic polynomial.
4. Let  $T$  be a linear operator on  $V$ . Show that range of  $T$  and null space of  $T$  are invariant under  $T$ .
5. **True or False.** Justify. "Every inner product space is a metric space".
6. Let  $W$  be a subspace of  $\mathbb{R}^4$  consisting of all vectors which are orthogonal to both  $x = (1, 0, -1, 1)$  and  $y = (2, 3, -1, 2)$ . Find a basis for  $W$ .

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

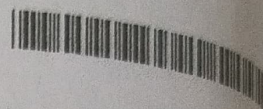
Unit – I

7. A) Define rank and nullity of a linear transformation. Let  $V$  and  $W$  be vector spaces over the field  $F$  and  $T$  be a linear transformation from  $V$  into  $W$ . Suppose  $V$  is finite dimensional then show that  $\text{rank}(T) + \text{Nullity}(T) = \dim V$ .  
B) Let  $T$  is a function from  $\mathbb{R}^2$  into  $\mathbb{R}^2$  defined by  $T(x, y) = (y, x)$ . Check whether  $T$  is a linear transformation or not ?

P.T.O.







8. A) Let  $V$  and  $W$  be Vector Spaces over the field  $F$  and let  $T$  is a linear transformation from  $V$  into  $W$ . If  $T$  is invertible then show that, the inverse function  $T^{-1}$  is a linear transformation from  $W$  onto  $V$ .

B) Give an example of a linear transformation, which is not onto and non-singular. Also give an example of a linear transformation, which is singular and not onto.

9. A) Let  $V$  and  $W$  are Vector Spaces over the field  $F$  and let  $T$  is a linear transformation from  $V$  into  $W$ . The null space of  $T^t$  is the annihilator or range of  $T$ . If  $V$  and  $W$  are finite dimensional then show that

i)  $\text{Rank}(T^t) = \text{Rank}(T)$

ii) The range of  $T^t$  is the annihilator of the null space of  $T$ .

B) Let  $A$  be an  $m \times n$  matrix over the Field  $F$ . Then show that row rank of  $A$  = column rank of  $A$ .

### Unit – II

10. State and prove Cayley-Hamilton Theorem.

11. A) Let  $V$  be finite dimensional vector space over a field  $F$  and let  $T$  be a linear operator on  $V$ . Then show that  $T$  is triangulable if and only if the minimal polynomial for  $T$  is the product of linear polynomials over  $F$ .

B) Let  $V$  be finite dimensional vector space over a field  $F$  and let  $T$  be a linear operator on  $V$ . Then show that  $T$  is diagonalizable if and only if the minimal polynomial for  $T$  has the form  $p = (x - C_1) \dots (x - C_k)$ , where  $C_1, \dots, C_k$  are distinct elements of  $F$ .

12. A) Find the minimal polynomial for the  $4 \times 4$  matrices

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

B) Find an invertible real matrix  $p$  such that  $P^{-1}AP$  are diagonal.

Where  $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$ .





Unit – III

13. A) Let  $T$  be a linear operator on a finite dimensional vector space  $V$  over the field  $F$ . Suppose that minimal polynomial for  $T$  decomposes over into a product of linear polynomials. Show that there is a diagonalizable operator  $D$  on  $V$  and a nilpotent operator  $N$  on  $V$  such that
- i)  $T = D + N$
  - ii)  $DN = ND$
- $D$  and  $N$  are uniquely determined by (i) and (ii) and each of them is a polynomial in  $T$ .
- B) If  $A$  is the companion matrix of a monic polynomial  $p$ , then show that  $p$  is the minimal and characteristic polynomial of  $A$ .
14. A) Let  $A$  be a complex  $3 \times 3$  matrices given by  $A = \begin{bmatrix} 2 & 0 & 0 \\ a & 2 & 0 \\ b & c & -1 \end{bmatrix}$ . Show that  $A$  is similar to diagonal matrix if and only if  $a = 0$ .
- B) Let  $V$  be the space of all  $n$ -time differentiable functions on an interval of real line which satisfying the differential equation  $\frac{d^n f}{dx^n} + a_{n-1} \frac{d^{n-1} f}{dx^{n-1}} + \dots + a_1 \frac{d^1 f}{dx^1} + a_0 f = 0$ , where the coefficients are complex numbers. Let  $D$  be the differential operator on  $V$ . What is the Jordan form for the differentiation operator on  $V$ ?
15. A) Define inner product space. Give an example of inner product space.
- B) State and prove polarization identities in real and complex cases.
- C) Show that every finite dimensional inner product has an orthonormal basis.