K22P 1602

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT1C02 : Linear Algebra

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Find a basis for a Vector Space $V = \{(x, y, z) \in R^3 / y = z + x\}$.
2. Let F be a Field and let T be a operator on R^3 defined by $T(x, y, z) = (x + 2y, x + y + z, 2y + 4z)$. Find the Matrix of T with respect to standard basis.
3. Show that similar matrices have same characteristic polynomial.
4. Let T be a linear operator on V. Show that range of T and null space of T are invariant under T.
5. **True** or **False**. Justify. "Every inner product space is a metric space".
6. Let W be a subspace of R^4 consisting of all vectors which are orthogonal to both $x = (1, 0, -1, 1)$ and $y = (2, 3, -1, 2)$. Find a basis for W.

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. A) Define rank and nullity of a linear transformation. Let V and W be vector spaces over the field F and T be a linear transformation from V into W. Suppose V is finite dimensional then show that rank (T) + Nullity (T) = dimV.

   B) Let T is a function from R^2 into R^2 defined by $T(x, y) = (y, x)$. Check whether T is a linear transformation or not ?

P.T.O.

8. A) Let V and W be Vector Spaces over the field F and let T is a linear transformation from V into W. If T is invertible then show that, the inverse function T^{-1} is a linear transformation from W onto V.

B) Give an example of a linear transformation, which is not onto and non-singular. Also give an example of a linear transformation, which is singular and not onto.

9. A) Let V and W are Vector Spaces over the field F and let T is a linear transformation from V into W. The null space of T^t is the annihilator or range of T. If V and W are finite dimensional then show that

i) Rank (T^t) = Rank (T)

ii) The range of T^t is the annihilator of the null space of T.

B) Let A be an $m \times n$ matrix over the Field F. Then show that row rank of A = column rank of A.

Unit – II

10. State and prove Cayley-Hamilton Theorem.

11. A) Let V be finite dimensional vector space over a field F and let T be a linear operator on V. Then show that T is triangulable if and only if the minimal polynomial for T is the product of linear polynomials over F.

B) Let V be finite dimensional vector space over a field F and let T be a linear operator on V. Then show that T is diagonalizable if and only if the minimal polynomial for T has the form

$p = (x - C_1) \ldots\ldots\ldots (x - C_k)$, where $C_1, \ldots\ldots C_k$ are distinct elements of F.

12. A) Find the minimal polynomial for the 4×4 matrices $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

B) Find an invertible real matrix p such that $P^{-1}AP$ are diagonal.

Where $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$.

Unit – III

13. A) Let T be a linear operator on a finite dimensional vector space V over the field F. Suppose that minimal polynomial for T decomposes over into a product of linear polynomials. Show that there is a diagonalizable operator D on V and a nilpotent operator N on V such that

i) T = D + N

ii) DN = ND

D and N are uniquely determined by (i) and (ii) and each of them is a polynomial in T.

B) If A is the companion matrix of a monic polynomial p, then show that p is the minimal and characteristic polynomial of A.

14. A) Let A be a complex 3 × 3 matrices given by $A = \begin{bmatrix} 2 & 0 & 0 \\ a & 2 & 0 \\ b & c & -1 \end{bmatrix}$. Show that A is similar to diagonal matrix if and only if a = 0.

B) Let V be the space of all n-time differentiable functions on an interval of real line which satisfying the differential equation $\frac{d^n f}{dx^n} + a_{n-1}\frac{d^{n-1} f}{dx^{n-1}} + \ldots\ldots\ldots + a_1 \frac{d^1 f}{dx^1} + a_0 f = 0$, where the coefficients are complex numbers. Let D be the differential operator on V. What is the Jordan form for the differentiation operator on V ?

15. A) Define inner product space. Give an example of inner product space.

B) State and prove polarization identities in real and complex cases.

C) Show that every finite dimensional inner product has an orthonormal basis.

____